Improvement of theoretical storm characterization for different climate conditions

Mª Jesús Martín Soldevilla , Melva Martín-Hidalgo , Vicente Negro ,
J.S. López-Gutiérrez , Paloma Aberturas

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A B S T R A C T

The different theoretical models related with storm wave characterization focus on determining the significant wave height of the peak storm, the mean period and, usually assuming a triangle storm shape, their duration. In some cases, the main direction is also considered. Nevertheless, definition of the whole storm history, including the variation of the main random variables during the storm cycle is not taken into consideration.

The representativeness of the proposed storm models, analysed in a recent study using an empirical maximum energy flux time dependent function shows that the behaviour of the different storm models is extremely dependent on the climatic characteristics of the project area. Moreover, there are no theoretical models able to adequately reproduce storm history evolution of the sea states characterized by important swell components.

To overcome this shortcoming, several theoretical storm shapes are investigated taking into consideration the bases of the three best theoretical storm models, the Equivalent Magnitude Storm (EMS), the Equivalent Number of Waves Storm (ENWS) and the Equivalent Duration Storm (EDS) models. To analyse the representativeness of the new storm shape, the aforementioned maximum energy flux formulation and a wave overtopping discharge structure function are used.

With the empirical energy flux formulation, correctness of the different approaches is focussed on the progressive hydraulic stability loss of the main armour layer caused by real and theoretical storms. For the overtopping structure equation, the total volume of discharge is considered. In all cases, the results obtained highlight the greater representativeness of the triangular EMS model for sea waves and the trapezoidal (*nonparallel sides*) EMS model for waves with a higher degree of wave development.

Taking into account the increase in offshore and shallow water wind turbines, maritime transport and deep vertical breakwaters, the maximum wave height of the whole storm history and that corresponding to each sea state belonging to its cycle's evolution is also considered. The procedure considers the information usually available for extreme waves' characterization. Extrapolations of the maximum wave height of the selected storms have also been considered. The 4th order statistics of the sea state belonging to the real and theoretical storm have been estimated to complete the statistical analysis of individual wave height.

1. Introduction

The recent detailed investigation of the representativeness of a proposed theoretical storm model described in the literature (Martín-Hidalgo et al., 2014) shows the influence of the approximation used to define geometric shape, duration and mean period. The study introduced two new theoretical storm models, referred to as Equivalent Triangle Magnitude Storm (ETMS) and the Equivalent Triangle Number of Waves Storm (ETNWS) and analysed different approaches to define the mean period variation in the theoretical storm evolution. Their results were compared with the proposed theoretical storm models including the most popular Equivalent Triangle Storm model (ETS) drawn up by Boccotti (2000), the generalization of the ETS model, called Equivalent Power Storm (EPS) proposed by Fedele and Arena (2009), and the modified ETS model, here referred to as the Equivalent Triangle Duration Storm (ETDS) used by Corbella and Stretch (2012a) in coastal studies.

Using the maximum momentum flux model (Melby and Kobayashi, 2011), the accuracy of the different theoretical storm models was evaluated, comparing the progressive damage to the breakwater's main armour caused by the real and theoretical storms. The results obtained show that the ETS model proposed by Boccotti (2000) is acceptable to reproduce the damage progression of maritime structures facing typical sea storms. However, the model underestimates damage for sea states heavily influenced by swell wave components. The EPS model proposed by Fedele and Arena (2009), overestimates the damage irrespective of the storm's degree of development. The other triangular models, ETDS, ETMS and ETNWS correctly reproduce the evolution of and final structural damage. However for storms associated to swell wave components, the triangular storm shape is not appropriate and taking other theoretical storm shapes into consideration is required. The study also considered different approaches to define the mean period of the different sea states that make up the storm, emphasizing the good performance of Martín Soldevilla et al. (2009) with all storm types.

The much earliest of these studies is the Castillo et al. (1977) work. The author, focussed on the maximum wave height estimation, emphasizes the importance of considering the storm shape for extrapolation. Due to the lack of the measured and numerical long time series wave data, the author based his investigation on ship observations and, assuming several empirical and subjective approaches, considers that the increasing and decreasing storm's evolution are symmetrical and equal to 4 h. The decrease and increase storm sides are adjusted to a fourth order polynomial.

Storm shape patterns, copula functions and also magnitude concepts, are aspects considered in the ROM 1.0-09 Recommendations for the Project Design and Construction of Breakwaters (2009). This paper's intention is to make up for the lack of a versatile procedure to determine the load cycle characterization for the required return period.

The maximum wave height is defined in order to complete the wave characteristics making up the storm's history, i.e., determining it in the structural resistance of vertical breakwaters, offshore platforms, marine wind turbines, overtopping and ships' navigation. In the most frequent climatic conditions, the linear superposition of a narrow band wave spectrum is the mechanism responsible for the formation of extreme waves and the probability distribution for wave height follows a Rayleigh distribution (Longuet-Higgins, 1952); corrections due to finite spectral bandwidth by Boccotti (1989), Naess (1985), and Tayfun (1981). However, the evidence of freak wave generation in the real ocean such as the well known Draupner, or New Year wave, measured in 1995 at the Draupner North sea platform (Guedes Soares et al., 2003; Stansell et al., 2003) and, the occurrence of several serious accidents (i.e. a ship sinking or collapsing) related to such extreme rare waves, have led to re-analysing the wave mechanics and development of new sea wave models to process the effects of nonlinearity in individual wave distribution (Janssen, 2003; Onorato et al., 2001; Onorato et al., 2006; Yasuda et al., 1992; Yasuda et al., 1997). Most of them focus on a narrow band spectrum. A Stokes correction for nonlinearity in hard seas associated with a non-narrow band spectrum was obtained by Dawson (2004), using the wave theory of Stokes' fifth order. Their results were compared with experimental wave tank data and with the measurement reported by Mori et al. (2000). A complete review of different wave height distribution models was made by Tayfun and Fedele (2007) who compared the more relevant extension models showing that of the whole nonlinear extension considered, the quasi-deterministic form of the Naess (1985) model and the Dawson (2004) non-narrow band model, seem to best describe simulated non-linear wave heights. However analyses of measured data indicate that the wave heights observed are best described by the Boccotti (1989) approach and much further investigation is needed to unequivocally resolve the issue.

Assuming the real uncertainties related with knowledge of wave height distribution under non linear conditions, considering previous studies, and taking into consideration the wave characteristics of the project area, two different approaches have been used to define the most probable maximum wave height in deep water during storm conditions. The possibility of linear sea states belonging to storm history is also considered. The Kurtosis value is used to distinguish between linear and non-linear sea states.

2. Storm characterization

In order to assess a possible relationship between storm shapes and the maritime climate characteristics of the study area (i.e., fetch length, wind–wave persistence, etc.), the same two points used in the aforementioned paper, (Martín-Hidalgo et al., 2014) were used. One, SIMAR_10442072, located on the NW Spanish Peninsular coast is exposed to developed sea states (swell), whilst the other, SIMAR_2083039, on the NE Insular Spanish Mediterranean Spanish coast, is exposed to typical sea wave storms. The historical climate data used cover a total of 44 years (from 1958 to 2001) and belong to the hind cast (SIMAR-44) database of "Puertos del Estado" (State Ports).

2.1. Storm models

From the theoretical models discussed in previous papers on storms Martín-Hidalgo et al. (2014) the three that best reproduce damage have been selected, using the same adjusting criterion but generalizing to other geometric shapes. The main characteristics of the selected models are:

The Equivalent Duration Storm model, (EDS), (generalization of ETDS "Equivalent Triangular Duration Storm" proposed by Corbella and Stretch (2012a) and Corbella and Stretch (2012b), takes the equivalent height, H_{equiv}, as a shape height, and base, D, the duration associated to the time the real storm remains above H_{T}, (Corbella and Stretch, 2012a; Corbella and Stretch, 2012b), see Fig. 1.

The Equivalent Magnitude Storm model (EMS), (generalization of the "Equivalent Triangular Magnitude Storm" proposed by Martín-Hidalgo et al. (2014)). The shape height is obtained in terms of the equivalent height, H_{equiv}, and the base, the theoretical storm duration, D_{shape}, is established such that its magnitude (area describing

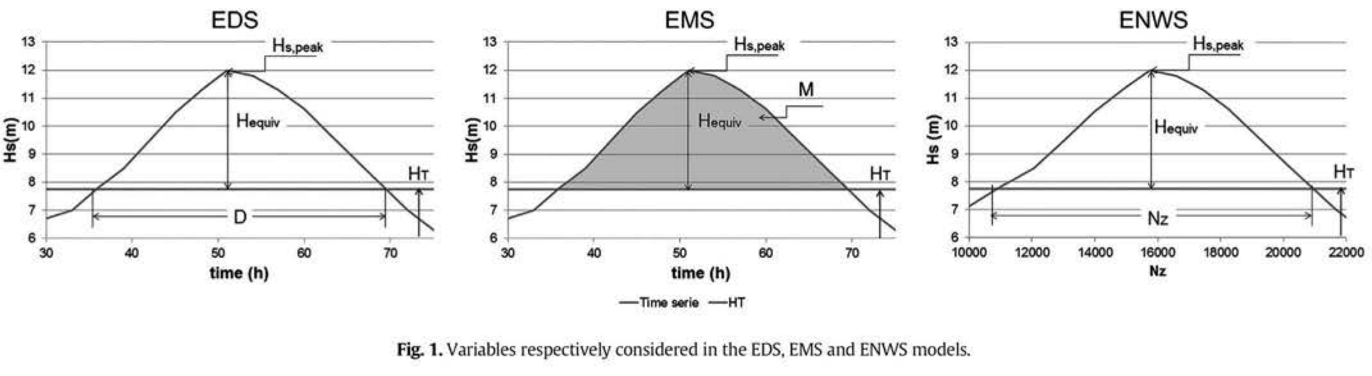

Fig. 1. Variables respectively considered in the EDS, EMS and ENWS models.

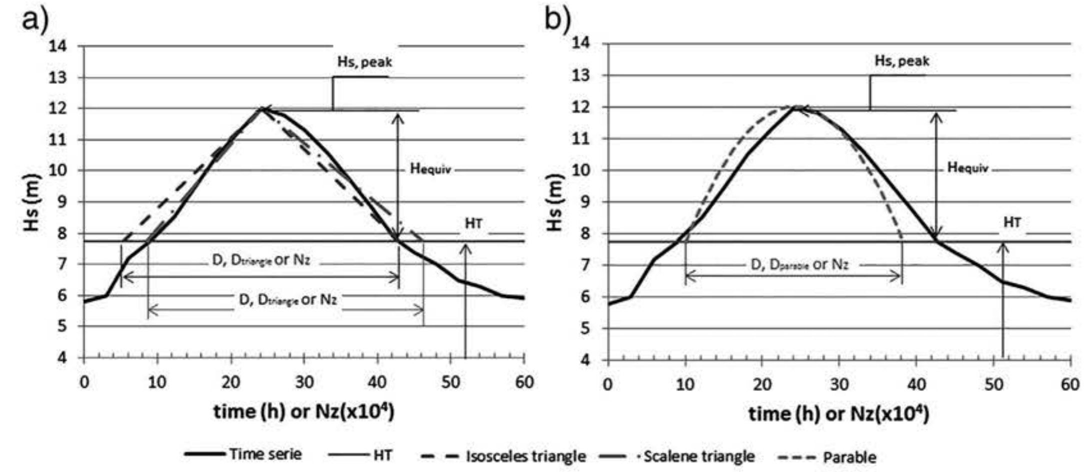

Fig. 2. Storm modelling shapes of a) isosceles & scalene triangles and b) parabola.

the storm history above H_T, which is a concept introduced by De Michele et al., 2007), equals the real storm (see Fig. 1).

The Equivalent Number of Waves Storm model, (ENWS), (generalization of the "Equivalent Triangular Number of Waves Storm" proposed by Martín-Hidalgo et al. (2014), the shape height is adjusted as a function of its equivalent height, H_{equiv}, and its base, the duration, in terms of the real storm number of waves, N_z, see Fig. 1.

2.2. Theoretical shapes

The geometric shapes we consider are: *the triangle* (isosceles and scalene), the *parabola*, referred to amongst others by Fedele and Arena (2009), *the trapezium*, parallel bases, (ROM 1.0-09 "Recommendations for the Project Design and Construction of Breakwaters") and *trapezoid*, non-parallel bases. Their definition is as follows:

The ***Isosceles Triangle*** is the shape used by the ETDS (Corbella and Stretch, 2012a; Corbella and Stretch, 2012b), ETMS and ETNWS models (Martín-Hidalgo et al., 2014). The storm peak is assumed to be at the midpoint of the storm history (see Fig. 2a.).

In the ***Scalene Triangle***, the peak is displaced (see Fig. 2a.) In order to obtain a simplified expression which may be used in the storm numerical simulation, the relative position ($X_{peak,i}$ where $X = D$ or Nz, $i = 1$ to n, where n is the total number of storms) of the maximum theoretical wave height $H_{s,peak}$ is obtained as a percentage of the total duration, (R_i) see Eq. (1).

$$R_i = X_{peak,i} / X_{total,i}, \tag{1}$$

where $X = D$ or Nz, $i = 1$ to n, where n is the total number of storms. Then the equivalent percentage R is established as:

$$R = (1/n)\Sigma_i^n R_i. \tag{2}$$

In the ***Parabola*** definition, the parabola vertex coincides with the storm peak, and the base, the storm duration (D, $D_{parabola}$ or N_z), (see Fig. 2b).

In order to define the trapezium or trapezoid shape, all storms recorded were transformed by escalating them to obtain the storm evolution pattern. The transformation involves dividing all equivalent significant wave heights, H_{equiv}, in the storm history by the equivalent significant wave height of the storm peak, $H_{Eq,peak}$ and the equivalent corresponding elapse time by the total duration. Thus, an equivalent storm is obtained, characterized by a significant wave height ($H_{Eq,peak,St} = 1$ m) and an equivalent duration, or number of waves ($D_{St} = 1\ h$, $N_{zSt} = 10^4$). The transformation scheme is shown in Fig. 3.

The average of the transformed areas, M_{RealSt}, is considered in order to set the ***equivalent trapezium pattern*** (parallel bases). Hence, the unknown smallest base b_S, is obtained by using the expression of the

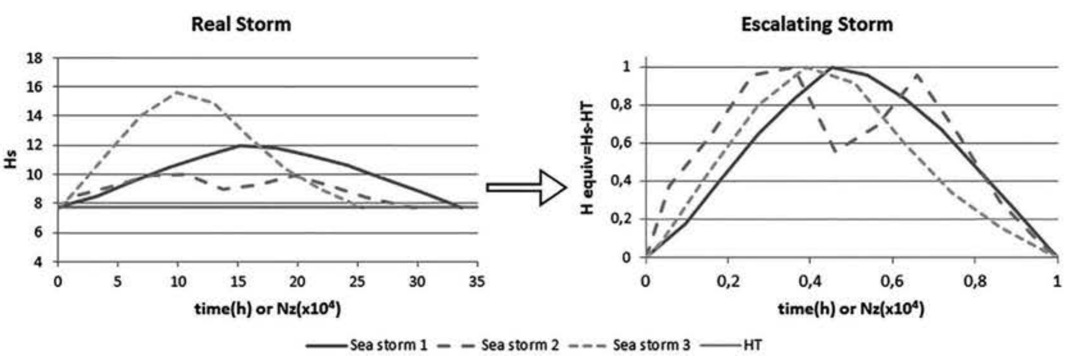

Fig. 3. Storm transformation.

standard storm magnitude (see Eq. (3)), characterized by a longest base $b_L = 1$ and a height $h = 1$.

$$M_{Escalating\ Real} = (1/2)(b_L + b_S)h, \tag{3}$$

Finally, the duration for the EMS model, $D_{Trapezium}$, is the resultant of the following equation:

$$M_{real,i} = (1/2)(1+b)H_{eq,i}D_{Trapezium,i} \tag{4}$$

where $i = 1$ to n, where n is the total number of storms.

The same transformation is applied to define the ***trapezoid*** shape, (nonparallel sides), and using the corresponding equivalent storm, the average positions of all the equivalent trapezium storm vertices (abscissas, $y_{1,i}$, $y_{2,i}$, referred to duration or number of waves, and ordinates, $y_{1,i}$, $y_{2,i}$, $i = 1,n$ referred to the equivalent wave height), the positions of the first $(\bar{x}_1, \bar{y}_1)$, and second, $(\bar{x}_2, \bar{y}_2)$, real storm peaks, are calculated (see Fig. 4a):

$$\left(\bar{x}_1 = (1/n)\cdot\sum_i^n x_{1,i}; \bar{y}_1 = (1/n)\cdot\sum_i^n y_{1,i};\right)\cdot\left(\bar{x}_2 = (1/n)\cdot\sum_i^n x_{2,i}; \bar{y}_2 = (1/n)\cdot\sum_i^n y_{2,i}\right) \tag{5}$$

where $i = 1$ to n, where n is the total number of storms.

Once the equivalent trapezoid pattern has been obtained, (referred to the duration or number of waves), the vertices of each storm can be obtained as a function of the equivalent trapezoid base pattern. ($D_{Trapezoid,i}$, $N_{z,i}$) (see Fig. 4b). Having established the positions of the theoretical storm pattern and considering the magnitude of the actual storm, $M_{real,i}$, and the corresponding significant wave height of the peak, $H_{s,peak}$, the length (duration, $D_{Trapezoid,i}$, or number of waves, $N_{z,i}$) corresponding to the untransformed theoretical storm in the EMS model, is obtained using the following equation:

$$M_{real,i} = (1/2)H_{eq,i}\,D_{\mathrm{Trapezoid},i}\left(\bar{x}_1\bar{y}_1 + (\bar{y}_1 + \bar{y}_2)\left(\bar{x}_2 - \bar{x}_1\right) + \bar{y}_2\left(1 - \bar{x}_2\right)\right) \tag{6}$$

where $i = 1$ to n, referred to the total (n) number of storms.

2.3. Sea state's mean period

Having defined the variables that characterize the extreme event, a single statistical characterization has been carried out. Here the multivariate characterization employing copulas is used

In order to evaluate the damage progression of the breakwater against the real and theoretical storm models, the best approach considered in Martín-Hidalgo et al. (2014) and the Martín Soldevilla et al. (2009) approach, to define the mean period of each sea state making up the whole storm history has been considered. This approach was made up of copula functions. The characteristics of extreme multivariate events are analysed in terms of the significant wave height at the peak $H_{s,peak}$ and the concomitant mean period, $T_{m,peak}$, of each of the storms. Based on the Gumbel copula, the authors propose selecting the most likely period for each wave height from the theoretical joint density function. A logarithmic relationship amongst the return significant wave height, $(H_s)_{R\_P}$, and the corresponding most probable mean period $(T_{m\_mode})_{R\_P}$ was obtained:

$$(T_{m\_mode})_{R\_P} \approx a_{R\_P} + b_{R\_P}\ln(H_s)_{R\_P} \tag{7}$$

The parameters a_{R_P}, b_{R_P} are functions of the characteristic storms at the project location:

$$\text{SIMAR-1042072}: T_m = -7.7 + 8.4\ln Hs \tag{8}$$
$$\text{SIMAR-2083039}: T_m = 0.8 + 4.0\ln Hs$$

The variables are Hs–M for the EMS model. Having analysed the performance of the different copula families and the variables' characteristics, Gumbel, Clayton and Frank are the selected copulas. Moreover, the joint probability function and bivariate copula distribution are represented (see Fig. 5).

3. Expected maximum wave height

Taking into account the representativeness of the different wave height distributions proposed in literature, the information available in the synthetic historical waves databases for its application and the spectral characteristics of the climatic areas analysed in this study, the Dawson (2004) model has been used to reproduce the maximum wave height of the typical non narrow bandwidth storms of the Mediterranean area. For the northwest Spanish coast facing more developed narrow band sea states, the predictions of Boccotti (1989) and Boccotti et al. (2013) wave height distribution was compared with the results of the Dawson (2004) model, to consider the sea states formed by swell with superimposed sea waves. The Rayleigh model was the approach employed for the linear sea states belonging to the storm history. The kurtosis, μ_4 formula is used to distinguish between linear and non linear sea states:

$$\mu_4 = 3 + 24k_0^2 m_0 \tag{9}$$

where: $k_0 = 2\cdot\pi/T_{m_{02}}$.

For the linear Rayleigh approach, the probability of exceedance $P(H_x)$ of the individual wave height scaled with statistical significant wave height can be expressed as:

$$P\left(H_x/H_{1/3} > h\right) = 1 - exp\left\{-\left[\frac{N_z}{exp\left(\left[H_x/(0.706\cdot H_{1/3}\right]^2\right)}\right]\right\} \tag{10}$$

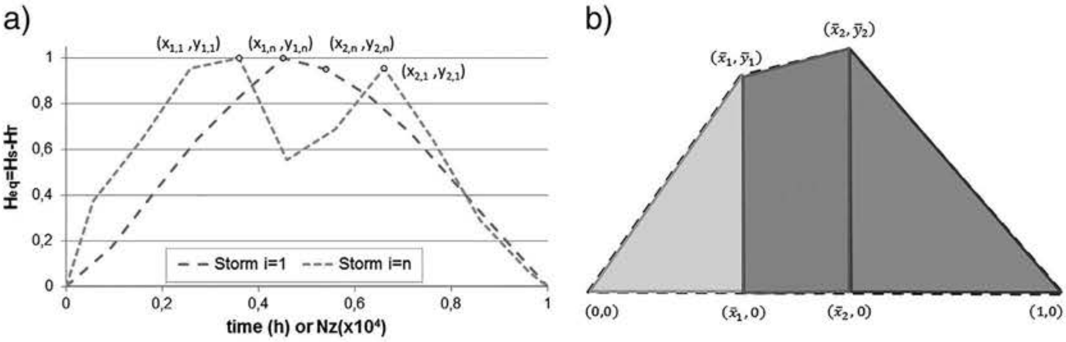

Fig. 4. a) Equivalent real storms $^{V_1}i(x_{1,i}, y_{1,i})$ and $^{V_1}i(x_{2,i}, y_{2,i})$ b) Transformed trapezium pattern storms.

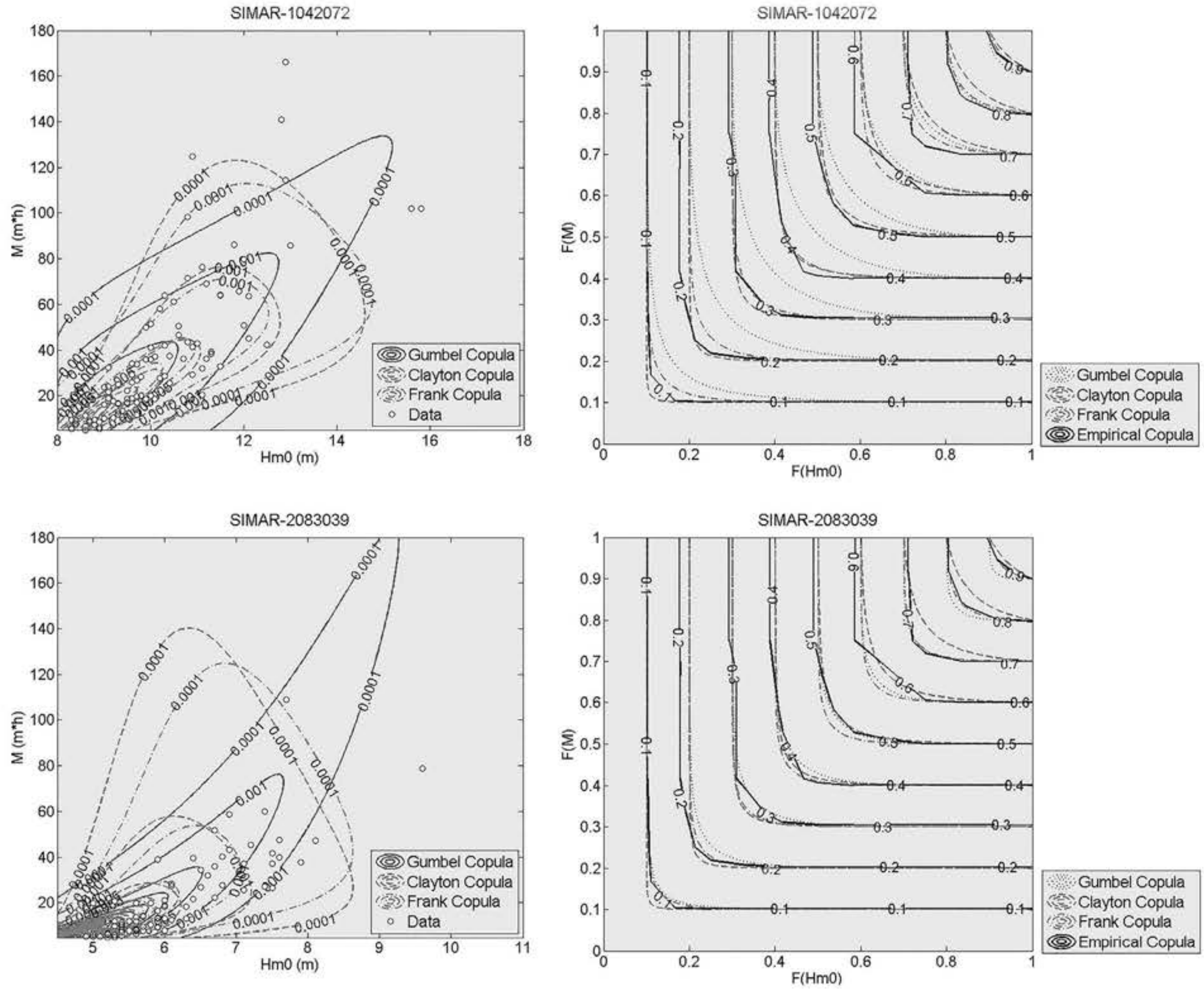

Fig. 5. Joint probability function and bivariate copula distribution of SIMAR-1042072 and of SIMAR-2083039.

where *Nz* is the number of waves of the sea state and $H_{1/3}$ the statistical significant wave height estimated from the available information as $H_{1/3} \cong 3.89\sqrt{m_0}$.

In the Dawson model, the probability of an exceedance function of the individual wave height H_x scaled with a zero spectral moment, m_0, takes the form:

$$P(H_x/m_0>h) = \ exp\left(-2\propto_D^2 h^2\right)\left[1-\frac{3}{16}\,\mu_D^2 h^2-\frac{107}{3072}\mu_D^4 h^4\right] \qquad (11)$$

where $a_D = m_0/H_{1/3}, \mu = \acute{\omega}\cdot m_0/g, \acute{\omega} = 2\pi/T$, is the steepness parameter, and $H_{1/3,}$ and $T\,'$ represent the significant wave height and mean zero up-crossing period derived from actual measurements.

And for the Boccotti approach the corresponding distribution function is given by:

$$P(Hx>h) = \ \exp\frac{1}{4(1+\varphi^*)}\frac{h^2}{m_0} \qquad (12)$$

where φ^*, called by the author the narrow *bandedness* parameter, is the complement of the bandwidth parameter that takes values between $0.65 < \varphi^* < 0.75$ in typical developed waves. For wind waves superimposed on swells, a wider spectrum is obtained and φ^* takes a lower value ($\varphi^* \approx 0.60$).

In all cases, the number of waves, $N_{Z,}$ of each of the sea states that make up the real and the theoretical EMS storm, is determined by calculating the probability of exceedance as

$$P\left(H_y>h\right) = \ 1/(N_z+1)\left\{\begin{array}{ll} H_y = \dfrac{H_x}{m_0} & for\ \text{Rayleigh and Dawson approach} \\ H_y = H_x & \text{for Bocotti expression}\end{array}\right\} \qquad (13)$$

where $N_Z =$ sea state duration time (3600 s)/sea state mean period (Real and Theoretical).

Given this probability, and a range of wave heights between H_s and $4H_s$, iterations were carried out by selecting the wave height that produces the probability of exceedance previously established in terms of N_Z

The comparisons of all the maximum wave heights estimated from the theoretical and real storms are shown in Fig. 6a & b for respectively the North-Atlantic and Mediterranean Seas.

Due to the good performance of the *geometrical–theoretical storm model*, the corresponding maximum wave height practically coincides. On the north coast of Spain where two approaches have been used, selecting the worst conditions is recommended. The H_{max} corresponding to each sea state belonging to the most energetic information point is presented in Fig. 7 for respectively the North-Atlantic and Mediterranean Seas.

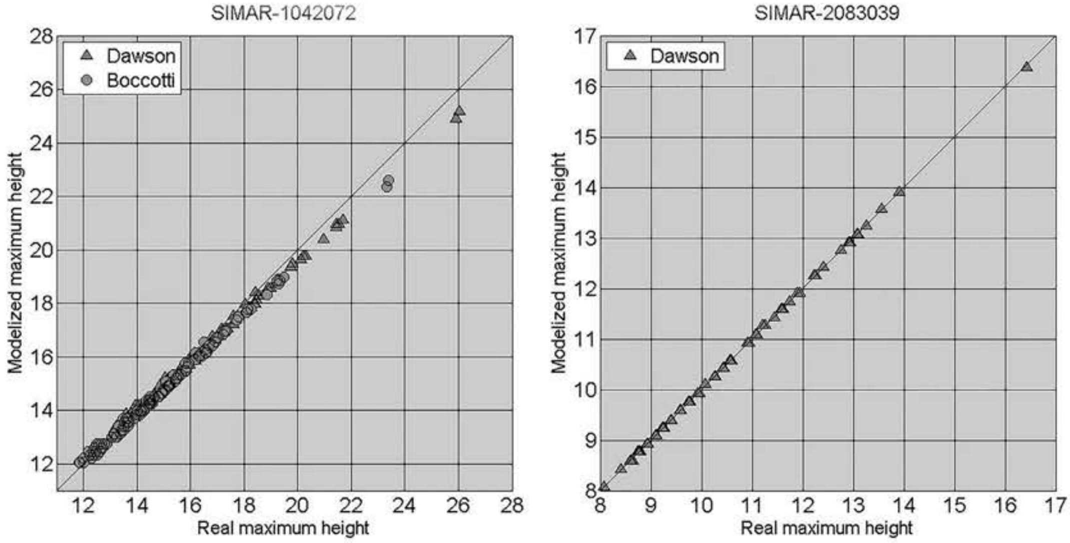

Fig. 6. Theoretical vs. empirical maximum wave heights of the severest storm. SIMAR-1042072 (North-Atlantic coast) and SIMAR-283099 (Mediterranean Sea).

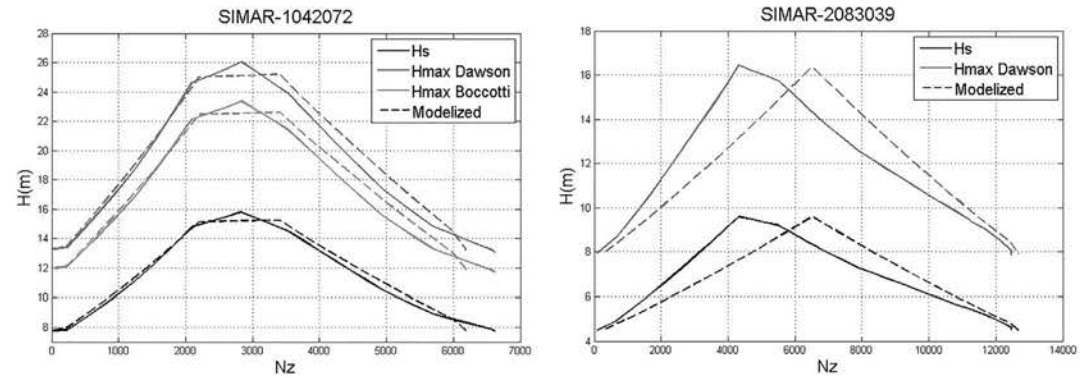

Fig. 7. Theoretical vs empirical maximum wave height of severest storm. a) SIMAR-1042072 (North Atlantic coast) b) SIMAR-2083039 (Mediterranean Sea).

Extrapolations for different return periods of the maximum wave height (using the Dawson and Boccotti approaches) of each storm have been carried out considering the 3-parameters Weibull distribution for the maximum wave height, and Poisson for the number of storms. The results, as seen in Fig. 8 show that the maximum return wave height resulting from the Boccotti model is 2 m lower than the Dawson's, keeping an accurate agreement between theoretical and real estimations.

Finally, to complete the statistical analysis of the individual wave height, the 4th order wave statistic heights belonging to the peak of each sea storm have been obtained. The comparison amongst real and synthetic estimations is presented in Table 1, referred to the mean square error, MSE. The results show that for developed sea states, as that corresponding to the SIMAR-1042072 point is, the EMS model with the trapezoid shape has the smallest MSE. For preponderant wind–sea storms, typical in the SIMAR-2083039 location, the three models behave in a similar fashion. Furthermore, geometrical shape election is essential, with the scalene triangle and the isosceles triangle in second place, as they have the lowest MSE.

4. Representativeness of the theoretical storm in maritime structure response

The empirical formula proposed by Melby and Kobayashi (2011), based on the concept of the maximum wave momentum flux at the toe of the structure, is used for checking the representativeness of the theoretical storm resulting from the different approaches and final damage is analysed, as is its progression in the course of both the real and theoretical storms.

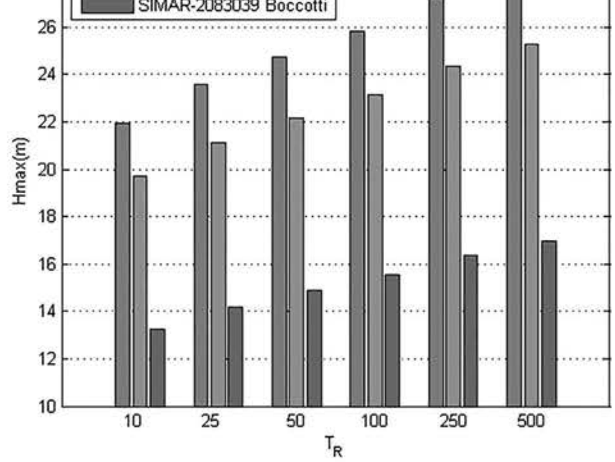

Fig. 8. Comparison of the maximum wave height return period (years), SIMAR-1042072 & SIMAR-2083039.

Table 1
The 4th order wave MSE for different approaches. The cases are in bold if their MSE is minor than 0.5.

		SIMAR-1042072				SIMAR-2083039			
	$H_{s,peak}$	15.80	15.60	12.90	12.90	9.600	8.100	7.900	7.700
EDS	Isosceles trian.	**0.363**	0.557	**0.119**	1.220	0.730	**0.130**	**0.044**	0.610
	Scalene trian.	**0.227**	**0.213**	0.562	1.000	**0.196**	0.041	0.030	**0.357**
	Parabola	1.904	0.975	**0.454**	**0.316**	0.777	**0.316**	**0.272**	1.151
	Trapezium	1.565	0.621	**0.354**	0.564	0.704	**0.204**	**0.103**	1.095
	Trapezoid	**0.452**	**0.135**	**0.470**	0.553	**0.164**	**0.069**	**0.197**	0.654
EMS	Isosceles trian.	**0.480**	**0.337**	**0.130**	0.612	0.638	**0.064**	**0.087**	**0.463**
	Scalene trian.	**0.329**	**0.200**	0.269	0.473	**0.126**	**0.055**	**0.087**	**0.249**
	Parabola	**0.433**	0.871	0.307	0.372	0.699	**0.247**	**0.092**	0.689
	Trapezium	**0.396**	**0.366**	**0.236**	**0.384**	0.615	**0.040**	**0.191**	0.846
	Trapezoid	**0.065**	**0.219**	**0.268**	**0.226**	**0.263**	**0.079**	**0.148**	0.771
ENWS	Isosceles trian.	**0.417**	**0.231**	**0.099**	0.986	0.513	**0.069**	**0.040**	0.637
	Scalene trian.	**0.218**	**0.155**	0.505	0.814	**0.090**	**0.038**	**0.055**	**0.371**
	Parabola	3.372	2.520	0.862	**0.292**	1.272	**0.468**	**0.317**	1.080
	Trapezium	1.866	1.299	**0.415**	0.533	0.819	**0.168**	**0.104**	1.132
	Trapezoid	2.098	0.975	**0.121**	0.691	1.062	**0.279**	**0.116**	1.276

To complete the accuracy analysis of the theoretical storm models, the modelled and the real total overtopping discharge volume resulting from a structure function are considered. For plane and smooth bermed slopes, the dimensionless ratio of overtopping wave discharge, Q, a formula suggested by Owen (1980), is used. Extending this formula to other kinds of breakwater is given by Van der Meer and Stam (1992):

$$Q = \frac{q}{\sqrt{g \cdot Hs^3}} = 8 \cdot 10^{-5} \cdot e^{3.10 \cdot \left(\frac{R_{u_{2\%}} - R_c}{Hs}\right)} \tag{14}$$

where q is the mean discharge (m^3/s per m), Rc is the crest freeboard and $R_{u_{2\%}}$ is the 2% run-up height, calculated using the de Waal and Van der Meer (1992) formula:

$$\begin{aligned} \frac{R_{u_{2\%}}}{Hs} &= 1.5 \cdot \gamma \cdot \xi_{op}; \qquad \xi_{op}<2(\text{plunging}) \\ \frac{R_{u_{2\%}}}{Hs} &= 3.0 \cdot \gamma; \qquad \xi_{op}>2(\text{surging}) \end{aligned} \tag{15}$$

where ξ_{op} is the Iribarren number and γ a total reduction factor taking roughness, berms, and wave obliquity attack into consideration.

This formula has limited application if the overtopping volume is large or the crest freeboard is similar to the 2% run-up height. However this aspect has no effect in our comparison due to the freeboard considered for the structure and the wave characteristics of the storm analysed.

Fifteen theoretical storms resulting from considering three storm models (EDS, EMS, ENWS), five geometric shapes for the period of the copula function proposed by Martín Soldevilla et al. (2009) were analysed for each SIMAR point studied, see Fig. 9.

The representativeness of the different approaches is analysed by reproducing the history of real storms and the corresponding theoretical ones in the damage progression model proposed by Melby and Kobayashi (2011) and the overtopping evolution formula proposed by Van der Meer and Stam (1992). Both failure modes were analysed taking into consideration the significant wave height of the storms and the corresponding three highest waves belonging to the peak storm.

The accuracy of the final main armour damage and overtopping volume discharge obtained with the different approaches is analysed by comparing the corresponding results to the theoretical and real storms in terms of the correlation coefficient, R^2, the mean squared error of the whole storm *MSE*, and mean squared error considering only the three highest, $MSE_{-3rd-O.S}$: (see Table 2).

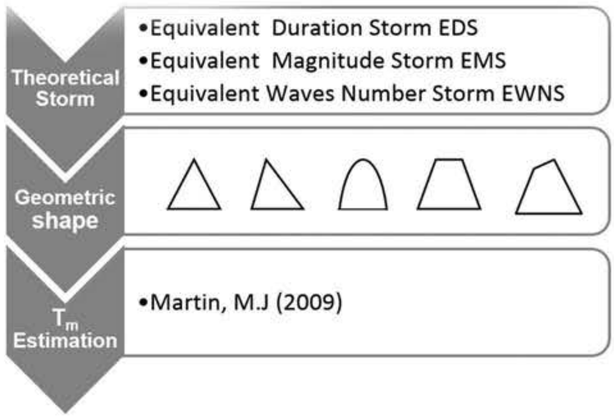

Fig. 9. Scheme of the approaches used.

The results obtained show a similar performance for the two structural functions (final damage and overtopping volume). The EMS model gives better results than EDS and ENWS, with correlation coefficients higher than the other models, and, besides, the MSE and mean squared error of the three maximum significant wave heights at the peak are smaller if the trapezoid and triangular storm shapes are considered.

The most simplistic method, the observation of real and theoretical damage dispersion, was also considered as proof of its correctness. Using the EMS model, the accuracy of different geometric shape in different climatic conditions is shown in Figs. 10 & 11. The deviation of damage evolution during the storm is presented, for the most energetic storms, at each SIMAR point studied in Fig. 12.

For developed sea states, the trapezoid at the SIMAR-1042072 point is the better storm shape because the theoretical overtopping volume discharge, and the final and main armour layer damage evolution is similar to the real ones, even for the highest storms. For preponderant wind–sea storms, using SIMAR-2083039 data, the storm shape which best fits is the triangle. The scalene triangle obtains slightly better results than the isosceles triangle. In all cases, the parabola and regular trapezium overestimate the damage.

Considering the model which achieves the best accuracy, a compact 3D damage progression representation has been drawn up (see Fig. 13). The probability of every damage level is the volume of the joint probability below every iso-damage (see Table 3).

5. Conclusions

The models proposed for characterizing the storm history display good performance although the Equivalent Magnitude Storm model, EMS, generally gets the best results for any kind of storms (predominant sea, swell or both). The Equivalent Duration Storm model, EDS and the Equivalent Number of Waves Storm model, ENWS, also give a good performance, however, depending on the characteristics of the storm, they tend to overestimate or underestimate damage.

The performance of the different theoretical shapes analysed is a function of the relative influence of sea and swell wave components in the project area. The triangle is recommended for typical sea storms whereas the trapezoid shape is much more appropriate for more developed storm conditions. The parabola and trapezium shapes overestimate damage in all kinds of storms. The results of the two triangles considered (isosceles & scalene) are practically the same but the isosceles is recommended due to its greater simplicity.

Taking into consideration that synthetic significant wave height coming from numerical spectral models does not provide any

Table 2
Summary of parameters for the different approaches considered.

Model	Shape	SIMAR-1042072						SIMAR-2083039					
		Damage			Overtopping			Damage			Overtopping		
		R2	MSE	MSE 3rd_OS	R2	MSE	MSE 3rd_OS	R2	MSE	MSE 3rd_OS	R2	MSE	MSE 3rd_OS
EDS	Isosceles trian.	0.941	0.100	0.510	0.910	864	2313	0.966	0.052	0.159	0.954	673	932
	Scalene trian.	0.941	0.100	0.512	0.910	864	2308	0.966	0.052	0.160	0.954	673	935
	Parabola	0.929	0.245	4.989	0.888	2637	46,176	0.888	0.359	6.275	0.865	4161	39,128
	Trapezium	0.925	0.259	5.821	0.902	2139	40,107	0.920	0.222	3.413	0.914	2099	11,557
	Trapezoid	0.977	0.054	0.296	0.957	635	3644	0.969	0.063	0.469	0.945	1111	3667
EMS	Isosceles trian.	0.984	0.029	0.384	0.990	118	1240	0.992	0.012	0.093	0.997	51	144
	Scalene trian.	0.984	0.029	0.385	0.990	118	1236	0.992	0.012	0.094	0.997	51	144
	Parabola	0.980	0.053	0.959	0.986	221	2963	0.962	0.091	2.054	0.993	129	2242
	Trapezium	0.960	0.120	2.582	0.968	561	9341	0.956	0.106	1.811	0.982	337	1614
	Trapezoid	0.992	0.017	0.030	0.993	101	89	0.993	0.013	0.177	0.996	64	557
ENWS	Isosceles trian.	0.945	0.102	0.176	0.926	789	410	0.966	0.061	0.268	0.953	800	1767
	Scalene trian.	0.946	0.101	0.175	0.926	789	410	0.965	0.061	0.269	0.953	800	1767
	Parabola	0.902	0.379	8.478	0.845	4343	89,908	0.848	0.577	11.501	0.806	7520	100,512
	Trapezium	0.920	0.288	6.867	0.892	2507	54,366	0.906	0.285	5.789	0.896	2846	33,922
	Trapezoid	0.968	0.082	0.968	0.949	836	11,742	0.955	0.101	1.405	0.928	1665	16,145

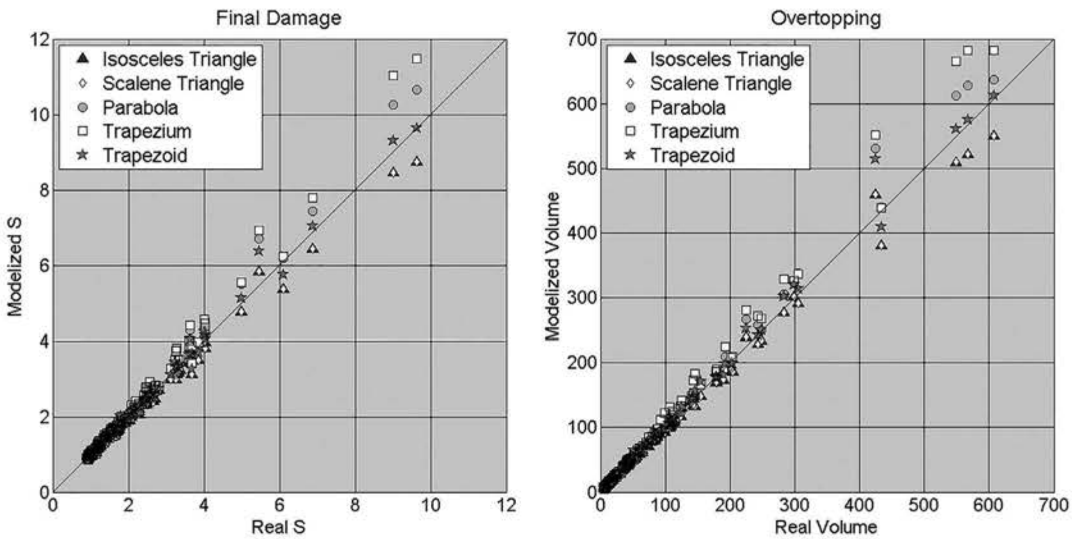

Fig. 10. Final Damage structure and total overtopping volume (m^3per m) caused by the real and theoretical storm model, SIMAR-1042072.

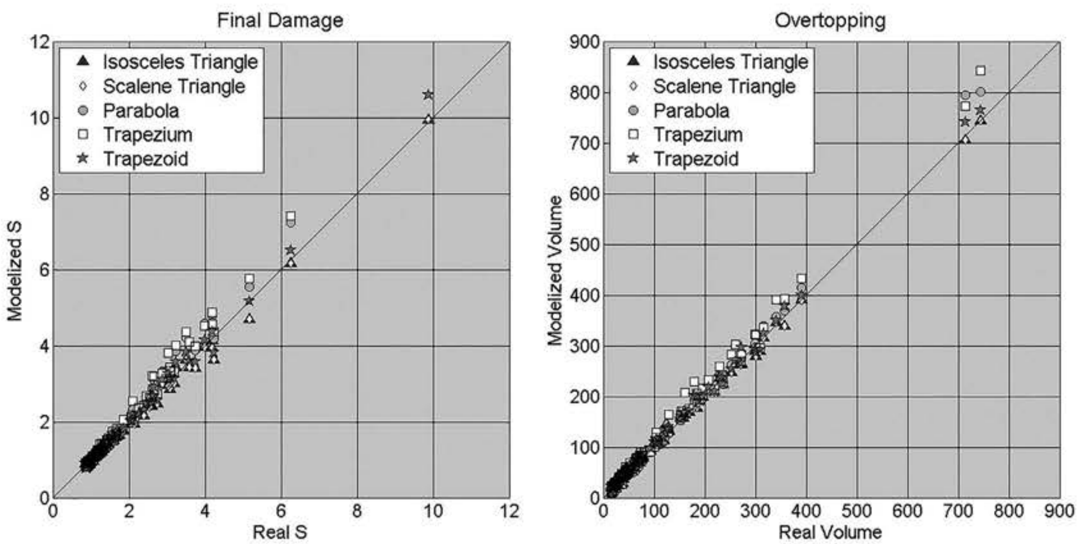

Fig. 11. Final damage structure and total overtopping volume (m^3per m) caused by the real and theoretical storm model, SIMAR-2083039.

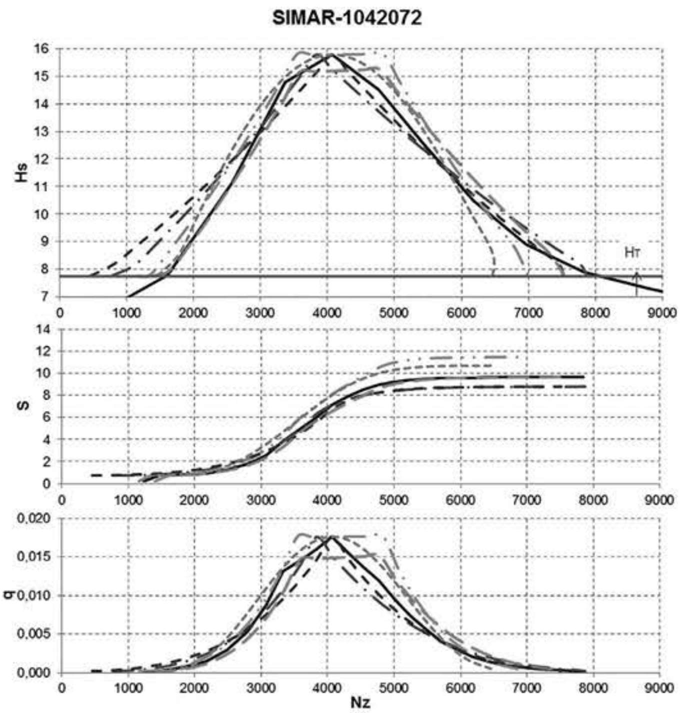

Fig. 12. Damage evolution and overtopping caused by real and theoretical storm models, SIMAR-1042072 and SIMAR-2083039.

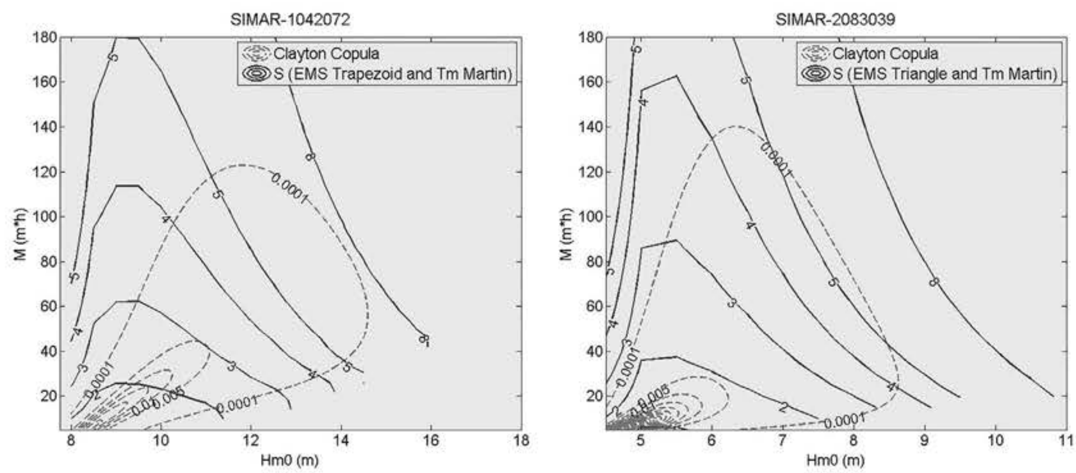

Fig. 13. Joint probability and theoretical damage level.

Table 3
Damage level probability.

S	SIMAR-1042072	SIMAR-2083039
0.5	0.99	1.00
1.0	0.99	0.99
1.5	0.88	0.60
2.0	0.67	0.47
2.5	0.51	0.40
3.0	0.39	0.36
3.5	0.31	0.34
4.0	0.25	0.32
5.0	0.18	0.31
6.0	0.15	0.30
7.0	0.14	0.30
8.0	0.13	0.29

information of the wave heights distribution of each sea state and the interest, mainly in vertical breakwaters design, of the knowledge of the maximum wave height, an extensive revision of the proposed wave height distribution models has been carried out. As conclusion of this analysis and assuming that much more investigations with measured time series are needed in order to define a general accuracy wave height distribution model, an approximation to determine the expected maximum wave height during storm is proposed. The procedure is based on kurtosis value and distinguishes amongst lineal and non-lineal sea conditions. Based on the wave characteristics of the project area, for nonlinear situations a classification amongst narrow and no-narrow band spectra is considered. For the former situation, Boccotti (1989) model is used and to the broad bandwidth spectra, the Dawson (2004) model is employed. For the lineal sea state belonging to the storm history the lineal Rayleigh model is taken up.

Using the appropriate storm pattern and the marginal and joint distribution functions, it is easy to define the complete evolution of the storm for the required lifetime, or return period, of the structure. To optimise the design of the structure, the probabilistic Level III verification of the failure modes with the corresponding simulated return storms is recommended. In this sense, consideration of other variables (wind speed and direction, currents, etc.) is feasible and essential for a complete structure optimization. To do that, previous effort has to be made in order to improve and develop more complete damage equations based on physical models and field data measurements.

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